OPTIMAL CONTROL FOR HALO ORBIT MISSIONS

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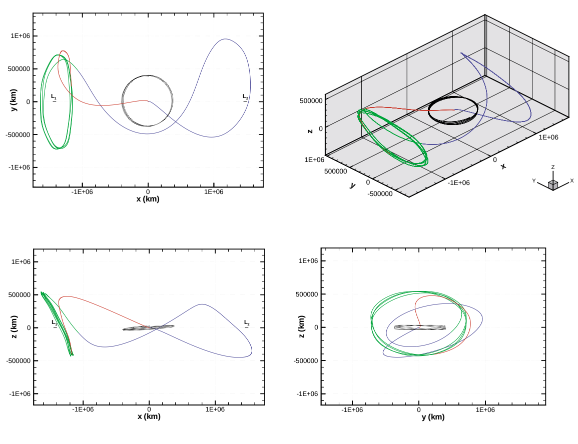
Abstract: This paper addresses the computation of the required trajectory correction maneuvers (TCM) for a halo orbit space mission to compensate for the launch velocity errors introduced by inaccuracies of the launch vehicle. By combining dynamical systems theory with optimal control techniques, we produce a portrait of the complex landscape of the trajectory design space. This approach enables parametric studies not available to mission designers a few years ago, such as how the magnitude of the errors and the timing of the first TCM affect the correction ΔV. The impetus for combining dynamical systems theory and optimal control in this problem arises from design issues for the Genesis Discovery mission being developed for NASA by the Jet Propulsion Laboratory.

Keywords: Optimal Control, Mission design, Dynamical systems.

1. INTRODUCTION AND BACKGROUND

The Genesis Mission Genesis is a solar wind sample return mission (see Lo et al [1998]). It is one of NASA's first robotic sample return missions and is scheduled for launch in January 2001 to a halo orbit in the vicinity of the L_1 Lagrange point, one of the five equilibrium points in the three body problem. L_1 is unstable and lies between the Sun and the Earth at roughly 1.5 million km from the Earth in the direction of the Sun. Once there, the spacecraft will remain for two years to collect solar wind samples before returning them to the Earth for study. Figure 1 shows the Genesis halo orbit and the transfer and return trajectories in a rotating frame. This rotating frame is defined by fixing the X-axis along the Sun-Earth line, the Z-axis in the ecliptic normal direction, and with the Y-axis completing a right-handed coordinate system. The Genesis trajectory



was designed using dynamical systems theory (see Howell et al [1997]). The three year mission, from launch all the way to Earth return, requires only a single small deterministic maneuver (less than 6 m/s) when injecting onto the halo orbit!

Halo Orbit Halo orbits are large three dimensional orbits shaped like the edges of a potato chip. The computation of halo orbits follows standard nonlinear trajectory computation algorithms based on parallel shooting.

The halo orbit, like the L_1 equilibrium point, is unstable. There is a family of asymptotic trajectories that departs from the halo orbit called the unstable manifold; similarly, there is a family of asymptotic trajectories which wind onto the halo orbit called the stable manifold. Each of these families form a two dimensional surface that is a twisted tubular surface eminating from the halo orbit. For Genesis, these manifolds are crucial for the mission design. The stable manifold, which winds onto the halo orbit, is used to design the transfer trajectory which delivers the Genesis spacecraft from launch to insertion onto the halo orbit (HOI). The unstable manifold, which winds off of the halo orbit, is used to design the return trajectory which brings the spacecraft and its precious samples back to Earth via a heteroclinic connection with L_2. See Koon et al [1999] for the current state of the computation of homoclinic and heteroclinic orbits in this problem.

Transfer Trajectory The transfer trajectory is designed using the following procedure. A halo orbit $H(t)$ is first selected, where t represents time. The stable manifold (W^S) of H consists of a family of asymptotic trajectories which take infinite time to wind onto H. Clearly, the exact asymptotic solutions cannot be found numerically and are impractical for space missions where the transfer time needs to be just a few months. Practically, there is a family of trajectories that lie arbitrarily close to W^S and that require just a few months to transfer between Earth and the halo orbit. A simple way to compute an approximation of W^S is based on Floquet theory.

In this paper, we will assume that the halo orbit, H(t), and the stable manifold M(t) are fixed and provided. We will not dwell further on their computation which is well covered in the references. Instead, let us turn our attention to the trajectory correction maneuver (TCM) problem.

TCM Problem The most important error in the launch of Genesis is the launch velocity error. The one sigma expected error is 7 m/s for a boost of approximately 3200 m/s from a circular 200 km altitude Earth orbit. Such an error is rather large because halo orbit missions are extremely sensitive to launch errors. Typical planetary launches can correct launch vehicle errors 7 to 14 days after the launch. In contrast, halo orbit missions must generally correct the launch error within the first day after launch. This correction maneuver is called TCM1, being the first TCM of any mission.

For orbits such as the Genesis transfer trajectory, the correction maneuver, ΔV for change in velocity, grows sharply in inverse proportion to the time from launch. For a large launch vehicle error, which is possible in Genesis' case, the TCM1 can quickly grow beyond the capability of the spacecraft's propulsion system.

The Genesis spacecraft, built in the spirit of NASA's new low cost mission approach, is very basic. This makes the performance of an early TCM1 difficult and risky. It is desirable to delay TCM1 as long as possible, even at the expense of expenditure of the ΔV budget. In fact, Genesis would prefer TCM1 be performed at 2 to 7 days after launch, or even later. However, beyond launch + 24 hours, the correction ΔV based on traditional linear analysis can become prohibitively high.

The desire to increase the time between launch and TCM1 drives one to use a nonlinear approach, based on combining dynamical systems theory with optimal control techniques. We explore two similar but slightly different approaches and are able to obtain in both cases an optimal maneuver strategy that fits within the Genesis ΔV budget of 450 m/s. (1) HOI technique: use optimal control techniques to retarget the halo orbit with the original nominal trajectory as the initial guess. (2) MOI technique: we target the stable manifold. Both methods yield good results.

2. OPTIMAL CONTROL FOR TRAJECTORY CORRECTION MANEUVERS

We now introduce the general problem of optimal control for dynamical systems. We start by recasting the TCM problem as a spacecraft trajectory planning problem. Mathematically they are exactly the same. Then we discuss the spacecraft trajectory planning problem as an optimization problem and highlight the formulation characteristics and particular solution requirements. Then the fuel efficiency caused by possible perturbation in the launch velocity and by different delay in TCM1 is exactly the sensitivity analysis of the optimal solution. The software we use is an excellent tool in solving this type of problem, both in providing a solution for the trajectory planning problem with optimal control and in studying the sensitivity of different parameters. COOPT is developed by the Computational Science and Engineering Group at University of California Santa Barbara (see Users' Guide [1999]).

We emphasize that the objective in this work is not to design the transfer trajectory, but rather to investigate recovery issues related to possible launch velocity errors. We therefore assume that a nominal transfer trajectory (corresponding to zero errors in launch velocity) is available. For the nominal trajectory in our numerical experiments in this paper, we do not use the actual Genesis mission transfer trajectory, but rather an approximation obtained with a restricted model.

Recast TCM as Trajectory Planning Problem We treat two distinct problems: (1) the halo orbit insertion (HOI) problem, in which we target the halo orbit, and (2) the stable manifold insertion (MOI) problem, in which we target the stable manifold associated with the halo orbit. Although different from a dynamical systems' perspective, the two problems are very similar once cast as optimization problems. In the HOI problem, a final maneuver (jump in velocity) is allowed at $T_{HOI} = t_{max}$, while in the MOI problem, the final maneuver takes place on the stable manifold at $T_{MOI} < t_{max}$ and no maneuver occurs at $T_{HOI} = t_{max}$. A halo orbit insertion trajectory design problem can be simply posed as:

> *Find the maneuver times and sizes to minimize fuel consumption (ΔV) for a trajectory starting near Earth and ending on the specified L_1 halo orbit at a position and with a velocity consistent with the HOI time.*

We assume that the evolution of the spacecraft is described by a generic set of six ODEs

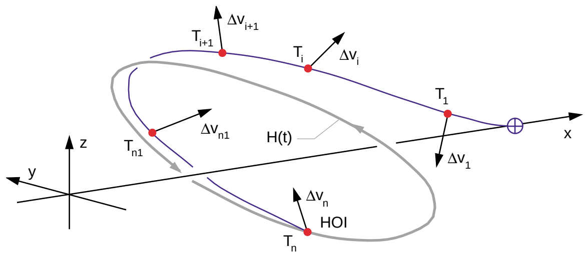
$$\mathbf{x}' = \mathbf{f}(t, \mathbf{x}), \qquad (2.1)$$

where $\mathbf{x} = [\mathbf{x}^p; \mathbf{x}^v] \in R^6$ contains both positions ($\mathbf{x}^p$) and velocities ($\mathbf{x}^v$). Eq. (2.1) can be either the Circular Restricted Three Body Problem (CR3BP) or a more complex model that incorporates the influence of the Moon and other planets. In this paper, we use the CR3BP model; other models will be investigated in future work.

In order to resolve the discontinuous nature of the resulting optimal control problem, the equations of motion (e.o.m.) are solved simultaneously on each interval between two maneuvers. Let the maneuvers $M_1, M_2, ..., M_n$ take place at times T_i, $i = 1, 2, ..., n$ and let $\mathbf{x}_i(t)$, $t \in [T_{i-1}, T_i]$ be the solution of Eq. (2.1) on the interval $[T_{i-1}, T_i]$ (see Figure 2). Continuity constraints at the position level are imposed at each maneuver, that is,

$$\mathbf{x}_i^p(T_i) = \mathbf{x}_{i+1}^p(T_i), \qquad i = 1, 2, ..., n-1. \qquad (2.2)$$

In addition, the final position is forced to lie on the halo orbit, that is, $\mathbf{x}_n^p(T_n) = \mathbf{x}_H^p(T_n)$, where the halo orbit is parameterized by the HOI time T_n. Additional constraints dictate that the first maneuver (TCM1) is delayed by at least a



prescribed amount $TCM1_{min}$ after launch, that is,

$$T_1 \geq TCM1_{min}, \qquad (2.3)$$

and that the order of maneuvers is respected,

$$T_{i-1} < T_i < T_{i+1}, \qquad i = 1, 2, ..., n-1. \qquad (2.4)$$

With a cost function defined as some measure of the velocity discontinuities

$$\begin{aligned} \Delta\mathbf{v}_i &= \mathbf{x}_{i+1}^v(T_i) - \mathbf{x}_i^v(T_i), \qquad i = 1, 2, ..., n-1, \\ \Delta\mathbf{v}_n &= \mathbf{x}_H^v(T_n) - \mathbf{x}_n^v(T_n), \end{aligned} \qquad (2.5)$$

the optimization problem becomes

$$\min_{T_i, \mathbf{x}_i, \Delta\mathbf{v}_i} f(\Delta\mathbf{v}_i), \qquad (2.6)$$

subject to the constraints in Eqns. (2.2)-(2.5). More details on selecting the form of the cost function are given in Section 3.

Launch Errors and Sensitivity Analysis In many optimal control problems, obtaining an optimal solution is not the only goal. The influence of problem parameters on the optimal solution (the so called sensitivity of the optimal solution) is also needed. In this paper, we are interested in estimating the changes in fuel efficiency (ΔV) caused by possible perturbations in the launch velocity (ϵ_0^v) and by different delays in the first maneuver (TCM1). As we show in Section 3, the cost function is very close to being linear in these parameters ($TCM1_{min}$ and ϵ_0^v). Therefore, evaluating the sensitivity of the optimal cost is a very inexpensive and accurate (especially in our problem) method of assesing the influence of different parameters on the optimal trajectory.

In COOPT, we make use of the Sensitivity Theorem (Bertsekas [1995]) for nonlinear programming problems with equality and/or inequality constraints. The influence of delaying the maneuver TCM1 is directly computed from the Lagrange multiplier associated with the constraint of Eq. (2.3). To evaluate sensitivities of the cost function with respect to perturbations in the launch velocity (ϵ_0^v), we must include this perturbation explicitly as an optimization parameter and fix it to some prescribed value through an equality constraint. That is, the launch velocity is set to

$$\mathbf{v}(0) = \mathbf{v}_0^{\text{nom}} \left(1 + \frac{\epsilon_0^v}{\|\mathbf{v}_0^{\text{nom}}\|} \right), \qquad (2.7)$$

where $\mathbf{v}_0^{\text{nom}}$ is the nominal launch velocity and

$$\epsilon_0^v = \epsilon, \qquad \epsilon \text{ given.} \tag{2.8}$$

The Lagrange multiplier associated with the constraint in Eq. (2.8) yields the desired sensitivity.

3. NUMERICAL RESULTS

Circular Restricted Three-Body Model As mentioned earlier, we use the equations of motion derived under the CR3BP assumption as the underlying dynamical model in Eq. (2.1). In this model, it is assumed that the primaries (Earth and Sun in our case) move on circular orbits around the center of mass of the system and that the third body (the spacecraft) does not influence the motion of the primaries. In a rotating frame and using nondimensional units, the equations of motion in the CR3BP model are

$$\ddot{x} = 2y + \frac{\partial U}{\partial x};\ \ddot{y} = -2x + \frac{\partial U}{\partial y};\ \ddot{z} = \frac{\partial U}{\partial z} \tag{3.1}$$

where $U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{d_\odot} + \frac{\mu}{d_\oplus}$, $d_\odot, d_\oplus$ are the distances between the spacecraft and the two primaries, and μ is the ratio between the mass of the Earth and the mass of the Sun-Earth system. In the above equations, time is scaled by the period of the primaries orbits ($T/2\pi$, where $T = 1$ year), positions are scaled by the Sun-Earth distance ($L = 1.49597927 \cdot 10^8$km), and velocities are scaled by the Earth's average orbital speed around the Sun ($2\pi L/T = 29.80567$km/s).

Choice of Cost Function. A physically meaningful cost function is

$$f_1(\Delta\mathbf{v}) = \sum_{i=1}^{n} \|\mathbf{v}_i\|. \tag{3.2}$$

This function is nondifferentiable when one of the maneuvers vanishes. This problem occurs at the first optimization iteration, as the intial guess transfer trajectory has a single nonzero maneuver at halo insertion. A differentiable cost function is

$$f_2(\Delta\mathbf{v}) = \sum_{i=1}^{n} \|\mathbf{v}_i\|^2. \tag{3.3}$$

Although this second cost function is more appropriate for the optimizer, it raises two new problems. Not only it is not as physically meaningful as the cost function of Eq. (3.2), but, in some particular cases, decreasing f_2 may actually lead to increases in f_1.

To resolve these issues, we use the following three-stage staggered optimization procedure:

(1) Starting with the nominal transfer trajectory as intial guess, and allowing initially n maneuvers, we minimize f_2 to obtain a first optimal trajectory, $\mathcal{T}_1^*$.

(2) Using $\mathcal{T}_1^*$ as initial guess, we minimize f_1 to obtain $\mathcal{T}_2^*$. It is possible that during this optimization stage some maneuvers can become very small. After each optimization iteration we monitor the feasibility of the iterate and the sizes of all maneuvers. As soon as at least one maneuver decreases under a prescribed threshold (0.1 m/s) at some fesible configuration, we stop the optimization algorithm.

(3) If necessary, a third optimization stage, using $\mathcal{T}_2^*$ as initial guess and f_1 as cost function is performed with a reduced number of maneuvers $\bar{n}$ (obtained by removing those maneuvers identified as 'zero maneuvers' in step 2).

Merging Optimal Control with Dynamical Systems Theory We present some results for the HOI problem and the MOI problems. A more indepth study will be given in a forthcoming publication. In both cases we investigate the effect of varying times for $TCM1_{min}$ on the optimal trajectory, for given perturbations in the nominal launch velocity. The staggered optimization procedure described above is applied for values of $TCM1_{min}$ ranging from 1 day to 5 days and perturbations in the magnitude of the injection velocity, ϵ_0^v, ranging from -7 m/s to $+7$ m/s. We present typical transfer trajectories, as well as the dependency of the optimal cost on the two parameters of interest. In addition, using the algorithm presented in Section 2, we perform a sensitivity analysis of the optimal solution. For the Genesis TCM problem, sensitivty information of first order is sufficient to characterize the influence of $TCM1_{min}$ and ϵ_0^v on the spacecraft performance.

The merging of optimal control and dynamical systems has been done through either (1) the use of the nominal transfer trajectory as a really accurate initial guess, or (2) the use of the stable invariant manifold.

Halo Orbit Insertion (HOI) Problem. In this problem we directly target the selected halo orbit with the last maneuver taking place at the HOI point. Using the optimization procedure described in the previous section, we compute the optimal cost transfer trajectories for various combinations of $TCM1_{min}$ and ϵ_0^v. In all of our computations, the launch conditions are those corresponding to a given nominal transfer trajectory with the launch velocity perturbed as described in Section 2.

As an example, we present complete results for the case in which the launch velocity is perturbed by -3 m/s and the first maneuver correction is delayed by at least 3 days. Initially, we allow for $n = 4$ maneuvers. In the first optimization stage, the second type of cost function has a value of $f_2^* = 1153.998$ (m/s)^2 after 5 iterations. This corresponds to $f_1^* = 50.9123$ m/s. During the second optimization stage, we monitor the sizes of all four maneuvers, while minimizing the cost function (3.2). After 23 iterations, the optimization was interrupted at a feasible configuration when at least one maneuver decreased below a preset tolerance of 0.1 m/s. The corresponding cost function is $f_1^{**} = 45.1216$ m/s with four maneuvers of sizes 33.8252 m/s, 0.0012 m/s, 0.0003 m/s, and 11.2949 m/s. In the last optimization stage we remove the second and third maneuvers and again minimize the cost function f_1. After 7 optimization iterations an optimal solution with $f_1^{***} = 45.0292$ m/s is obtained. The two maneuvers of the optimal trajectory have sizes of 33.7002 m/s and 11.3289 m/s and take place at 3.0000 and 110.7969 days after launch, respectively. Lagrange multipliers associated with the constraints of Eqs. (2.3) and (2.8) give the sensitivities of the optimal solution with respect to launching velocity perturbation, -10.7341 (m/s)/(m/s), and delay in first maneuver correction, 4.8231 (m/s)/days.

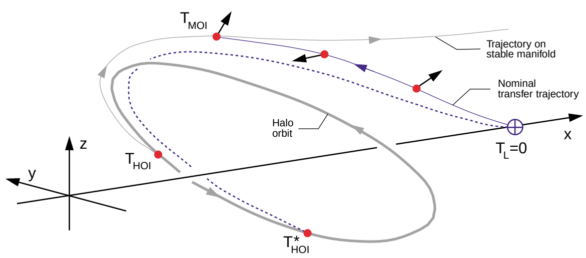
Launch Errors and Sensitivity Analysis. The staggered optimization procedure was applied for all values of $TCM1_{min}$ and ϵ_0^v in the region of interest. In a first experiment, we investigate the possibility of correcting for errors in the launch velocity using at most two maneuvers ($n = 2$). Numerical values of optimal cost as a function of these two parameters are given in Table 3. Except for the cases in which there is no error in the launch velocity (and for which the final optimal transfer trajectories have only one maneuver at HOI), the first correction maneuver is always on the prescribed lower bound $TCM1_{min}$. For all cases investigated, halo orbit insertion takes place at most 18.6 days earlier or 28.3 days later than in the nominal case ($T_{HOI} = 110.2$).

ϵ_0^v (m/s)	TCM1 (days)				
	1	2	3	4	5
-7	64.8086	76.0845	88.4296	99.6005	109.9305
-6	54.0461	67.0226	77.7832	86.8630	95.8202
-5	47.1839	57.9451	66.6277	74.4544	81.8284
-4	40.2710	48.8619	55.8274	62.0412	67.9439
-3	33.4476	39.8919	45.0290	49.6804	54.1350
-2	26.6811	30.9617	34.3489	37.3922	40.3945
-1	19.9881	22.2715	23.7848	25.2468	26.6662
0	13.4831	13.3530	13.4606	13.3465	13.2919
1	23.1900	21.9242	23.2003	24.4154	25.5136
2	26.2928	30.2773	33.3203	35.9203	38.3337
3	34.6338	38.8496	43.5486	47.7200	51.6085
4	41.4230	47.5266	53.9557	62.3780	65.1411
5	45.9268	56.2245	64.4292	75.0188	81.4325
6	53.9004	64.9741	76.6978	83.8795	95.2313
7	61.4084	75.9169	85.4875	98.4197	106.0411

Several conclusions can be drawn. First, for all cases that we investigated, the optimal costs are well within the ΔV budget allocated for trajectory correction maneuvers (450 m/s for the Genesis mission). The cost function is very close to being linear with respect to both $TCM1_{min}$ time and launch velocity error. Also, the halo orbit insertion time is always close enough to that of the nominal trajectory as not to affect either the collection of the solar wind or the rest of the mission (mainly the duration for which the spacecraft evolves on the halo orbit before initiation of the return trajectory).

Manifold Orbit Insertion (MOI) Problem. In the MOI problem the last nonzero maneuver takes place on the stable manifold and there is no maneuver to insert onto the halo orbit. A much larger parameter space is now investigated (we target an entire surface as opposed to just a curve) thus making the optimization problem much more difficult than the one corresponding to the HOI case. The first problem that arises is that the nominal transfer trajectory is not a good enough initial guess to ensure convergence to an optimal solution. To obtain an appropriate initial guess we use the following procedure: (1) We start by selecting an HOI time, T_{HOI}. This yields the position and velocity on the halo orbit. (2) With the above position and velocity as initial conditions, the equations of motion in Eq. (3.1) are then integrated backwards in time for a selected duration T_S along the stable manifold. This yields an MOI point which is now fixed in time, position, and velocity. (3) For a given value of $TCM1_{min}$ and with $\epsilon_0^v = 0$, and using the nominal transfer trajectory as initial guess, we use COOPT to find a trajectory that targets this MOI point, while minimizing f_1.

With the resulting trajectory as an initial guess and the desired value of ϵ_0^v we proceed with the staggered optimization presented before to obtain the final optimal trajectory for insertion on the stable manifold. During the three stages of the optimization procedure, both the MOI point and the HOI point are free to move (in position, velocity, and time) on the stable manifold surface and on the halo orbit, respectively. Taking the



launch time to be $T_L = 0$ and the HOI time (T^*_{HOI}) of the nominal transfer trajectory as a reference point on the halo orbit, we can investigate a given zone of the design space by an appropriate choice of the HOI point of our initial guess trajectory with respect to T^*_{HOI} (step 1 of the above procedure). That is, we select a value T_0 such that $T_{HOI} = T^*_{HOI} + T_0$. The point where the initial guess trajectory inserts onto the stable manifold is then defined by selecting the duration T_S for which the equations of motion are integrated backwards in time (step 2 of the above procedure). This gives a stable manifold insertion time of $T_{MOI} = T_{HOI} - T_S = T^*_{HOI} + T_0 - T_S$. Next, we use COOPT to evaluate these various choices for the initial guess trajectories (step 3 of the above procedure). A schematic representation of this procedure is shown in Figure 3.

Regions Best Suited for MOI Insertion. Using the values of $f_1(\Delta V)$, we can identify regions of the stable manifold that are best suited for MOI insertion. Examples are: (1) (*Region A*) MOI trajectories that insert on the halo orbit in the same region as the nominal transfer trajectory and which therefore correspond to initial guess trajectories with small T_0; (2) (*Region B*) MOI trajectories that have HOI points on the 'far side' of the halo orbit and which correspond to initial guess trajectories with halo insertion time around $T^*_{HOI} + 1.50$ ($T_0 = 1.50 \cdot 365/2\pi = 174.27$ days).

At first glance, trajectories in Region B might appear ill-suited to the Genesis mission as they would drastically decrease the duration for which the spacecraft evolves on the halo orbit (recall that design of the return trajectory dictates the time at which the spacecraft must leave the halo orbit). But for a typical MOI trajectory, all trajectories on the stable manifold asymptotically wind onto the halo orbit and are thus very close to the halo orbit for a significant time. This means that collection of solar wind samples can start much earlier than halo orbit insertion, therefore providing enough time for all scientific experiments before the spacecraft leaves the halo orbit.

After choosing a region of the stable manifold by selecting an initial guess trajectory, we perform a similar analysis as in the HOI problem. Consider correcting for perturbations in launch velocity by seeking optimal MOI trajectories in Region B, that is, on the far side of the halo from the Earth. For non-zero ϵ_0^v and $TCM1_{min}$, we compute an MOI initial guess trajectory with $T_0 = 1.50$ and $T_S = 0.75$ and then use the staggered optimization procedure to find an optimal MOI trajectory in this vicinity.

The results are given in Table 3. Note that the optimal MOI trajectories are close (in terms of the cost function f_1) to the corresponding HOI trajectories. Therefore, either method provides an excellent solution to the TCM problem.

4. CONCLUSIONS AND FUTURE WORK

This paper explored new approaches for autmomated parametric studies of optimal trajectory correction maneuvers for a halo orbit mission. Using the halo orbit insertion approach, for all the launch velocity errors and $TCM1_{min}$ considered we found optimal recovery trajectories. The cost functions (fuel consumption in terms of ΔV) are within the allocated budget even in the worst case

$TCM1_{min}(days)$	ϵ_0^v (m/s)	f_1 (m/s)
3	-3	45.1427
	-4	55.6387
	-5	65.9416
	-6	76.7144
	-7	87.3777
4	-3	49.1817
	-4	61.5221
	-5	73.4862
	-6	85.7667
	-7	99.3405
5	-3	53.9072
	-4	66.8668
	-5	81.1679
	-6	94.3630
	-7	109.2151

(largest $TCM1_{min}$ and largest launch velocity error).

Using the stable manifold insertion approach, we obtained similar results to those found using HOI targeted trajectories. The failure of the MOI approach to reduce the ΔV significantly may be because the optimization procedure (even in the HOI targeted case) naturally finds trajectories 'near' the stable manifold. We will investigate this interesting effect in future work.

For now, the main contribution of dynamical systems theory to the optimal control of recovery trajectories is in the construction of good initial guess trajectories in sensitive regions where optimizers have the greatest flexibility.

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