

Radiometry

Radiometry:

The study of the physical measurement of electromagnetic energy

Photometry:

The study of the perceptual response to light (electromagnetic energy)

Solid Angles

Angles

$$\theta = \frac{\textit{length}}{\textit{radius}}$$

2π radians in a circle

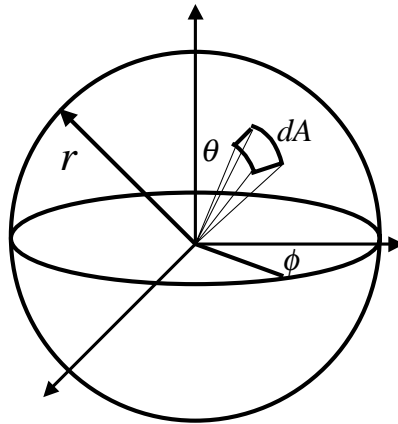
Solid Angles

$$\omega = \frac{\textit{area}}{\textit{radius}^2}$$

4π steradians in a sphere

If the area is not on a sphere, project it onto the unit sphere

Differential Solid Angles



$$dA = (r d\theta)(r \sin\theta d\phi)$$

$$= r^2 \sin\theta d\theta d\phi$$

$$d\omega = dA / r^2$$

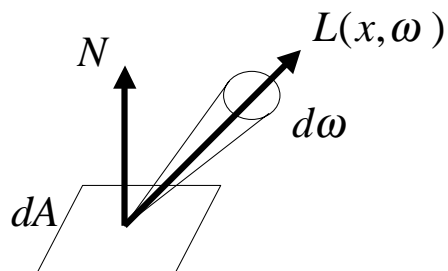
$$= \sin\theta d\theta d\phi$$

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Radiance

Radiance: Power per unit projected area perpendicular to the ray, per unit solid angle in the direction of the ray



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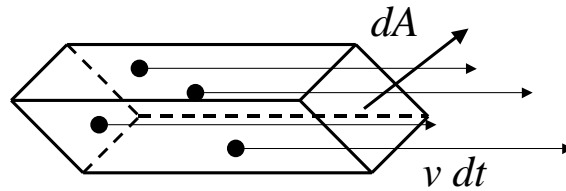
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Flux

Flux: Radiant Power

$$d\Phi = L(x, \omega) \cos\theta \, d\omega \, dA$$

Transport theory (how stuff flows)
views this as the flow of photons



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Properties of Radiance

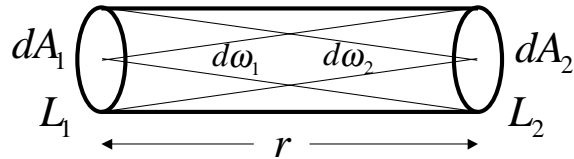
- 1) Fundamental quantity
 - all other quantities derived from it
- 2) Invariant along a ray
 - quantity used by ray tracers
- 3) Sensor response is proportional to radiance
 - eye/camera response depends on radiance

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Invariance of Radiance

$$d\Phi = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2$$



But: $d\omega_1 = dA_2 / r^2$ and $d\omega_2 = dA_1 / r^2$

Throughput: $T = d\omega_1 dA_1 = d\omega_2 dA_2 = \frac{dA_1 dA_2}{r^2}$

Therefore:

$$L_1 = L_2$$

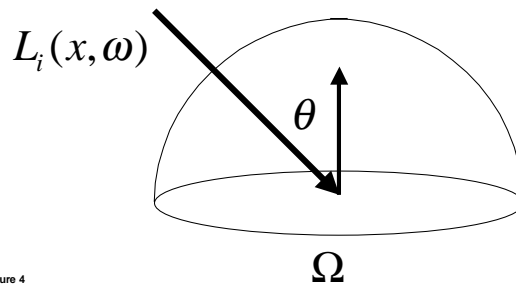
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Irradiance

Irradiance: Radiant power per unit area incident on a surface

$$E = \int_{\Omega} L_i(x, \omega) \cos\theta d\omega$$



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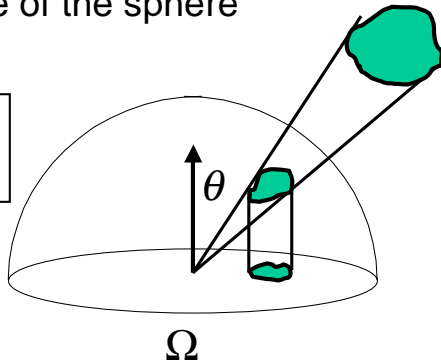
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Projected Solid Angle

Projected Solid Angle: $\cos\theta d\omega$

Projection of the differential area
(on the sphere's surface) onto
the base of the sphere

$$\int_{\Omega} \cos\theta d\omega = \pi$$



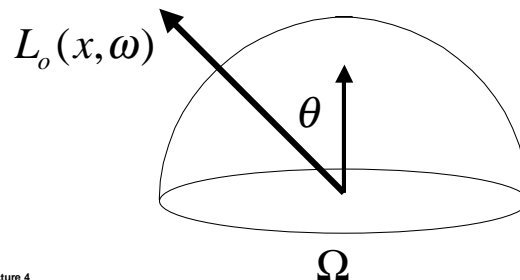
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Radiosity

Radiosity: Radiant power per unit area
exiting a surface

$$B = \int_{\Omega} L_o(x, \omega) \cos\theta d\omega$$



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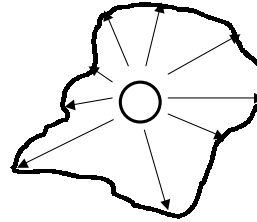
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Radiant Intensity

Radiant Intensity: Radiant power per solid angle of a point source

$$I(\omega) = \frac{d\Phi}{d\omega}$$

$$\Phi = \int_{\Omega} I(\omega) d\omega$$



For an isotropic point source: $I(\omega) = \frac{\Phi}{4\pi}$

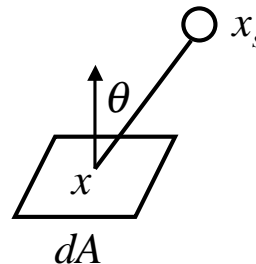
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Irradiance due to a Point Light

Irradiance on a differential surface due to an isotropic point light source

$$E = I(\omega) \frac{d\omega}{dA}$$
$$= \frac{\Phi}{4\pi} \frac{\cos\theta}{|x - x_s|^2}$$



Note the inverse squared distance fall-off

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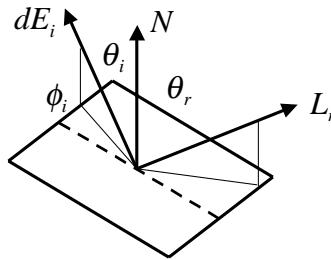
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BRDF

Bidirectional Reflection Distribution Function

$$f(x, \omega_i, \omega_r) = \frac{L_r(x, \omega_r)}{dE_i(x, \omega_i)}$$

The ratio of radiance in a reflected direction to the differential irradiance that created it



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Properties of the BRDF

Reciprocity

$$f(x, \omega_i, \omega_r) = f(x, \omega_r, \omega_i)$$

Anisotropy

$$f(x, \theta_i, \phi_i, \theta_r, \phi_r) \neq f(x, \theta_i, \phi_i + \phi, \theta_r, \phi_r + \phi)$$

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Reflectance Equation

The BRDF allows us to calculate outgoing light, given incoming light:

$$\begin{aligned}L_{r, \text{from } \omega_i}(x, \omega_r) &= f(x, \omega_i, \omega_r) dE_i(x, \omega_i) \\ &= f(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta d\omega_i\end{aligned}$$

Integrating over the hemisphere gives the reflectance equation:

$$L_r(x, \omega_r) = \int_{\Omega} f(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta d\omega_i$$

Reflection is linear

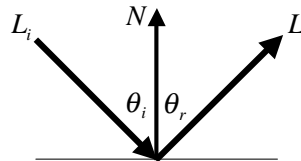
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Ideal Specular Reflection

Reflection is only in the mirror direction

$$\begin{aligned}\theta_r &= \theta_i \\ \phi_r &= \phi_i \pm \pi \\ L_r(\theta_r, \phi_r) &= L_i(\theta_i, \phi_i)\end{aligned}$$



$$f_{r, \text{mirror}} = \frac{\delta(\theta_r - \theta_i)}{\cos \theta_i} \delta(\phi_r - (\phi_i \pm \pi))$$

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Reflectance

Reflectance: ratio of reflected flux to incident flux

$$\begin{aligned}\rho &= \frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_r, \Omega_i} f_r(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i \cos \theta_r d\omega_i d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}\end{aligned}$$

Reflectance is always between 0 and 1
but depends on incident radiance distribution

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Reflectance

If incident distribution is uniform and isotropic

$$\begin{aligned}\rho &= \frac{\int_{\Omega_r, \Omega_i} f_r(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i \cos \theta_r d\omega_i d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &= \frac{\int_{\Omega_r, \Omega_i} f_r(\omega_i, \omega_r) \cos \theta_i \cos \theta_r d\omega_i d\omega_r}{\int_{\Omega_i} \cos \theta_i d\omega_i}\end{aligned}$$

This equation relates reflectance to the BRDF

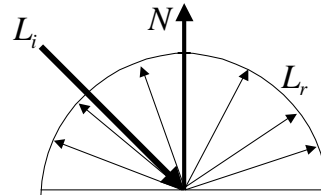
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Lambertian Diffuse Reflection

Reflection is equal in all directions

$f_{r,diffuse}(x, \omega_i, \omega_r)$
is constant



$$\begin{aligned}L(\omega_r) &= \int_{\Omega_i} f_{r,diffuse} L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,diffuse} \int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,diffuse} E\end{aligned}$$

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Lambertian Diffuse Reflection

Reflected radiance is independent of direction

Therefore the radiosity is simply:

$$\begin{aligned}B &= \int_{\Omega} L_{r,diffuse} \cos \theta d\omega \\ &= \pi L_{r,diffuse} \\ &= \pi f_{r,diffuse} E\end{aligned}$$

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Lambertian Diffuse Reflection

Reflectivity is constant:

$$\begin{aligned}\rho_{diffuse} &= \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i} \\ &= \frac{L_{r,diffuse} \int_{\Omega_r} \cos \theta_r d\omega_r}{E} \\ &= \frac{\pi L_{r,diffuse}}{E} \\ &= \pi f_{r,diffuse}\end{aligned}$$

$$\rho_{diffuse} = \frac{B}{E}$$

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Local Illumination

Depends only on the properties of the light source and the surface to be shaded

Has no notion of the surrounding environment:

- Assume all light reaches the surface
- Cannot handle inter-reflection and shadowing
- Cannot handle indirect illumination

The above must be handled by global illumination algorithms

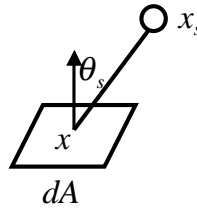
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Local Illumination

Given the irradiance from a point source:

$$E = \frac{\Phi}{4\pi} \frac{\cos\theta_s}{|x - x_s|^2}$$



The radiance from the point source is:

$$L_i(\omega_i) = \frac{\Phi}{4\pi} \frac{\delta(\theta_i - \theta_s) \delta(\phi_i - \phi_s)}{|x - x_s|^2}$$

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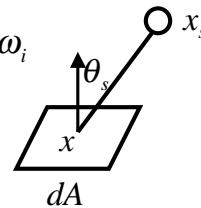
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Local Illumination

Substituting into the reflectance equation:

$$L_r(x, \omega_r) = \int_{\Omega} f(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos\theta \, d\omega_i$$

$$L_i(\omega_i) = \frac{\Phi}{4\pi} \frac{\delta(\theta_i - \theta_s) \delta(\phi_i - \phi_s)}{|x - x_s|^2}$$



The reflected radiance from the surface is:

$$L_r(\omega_r) = \frac{\Phi}{4\pi} \frac{\cos\theta_s}{|x - x_s|^2} f_r(\omega_s, \omega_r)$$

Lighting model used by most 3D graphics hardware

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Global Illumination

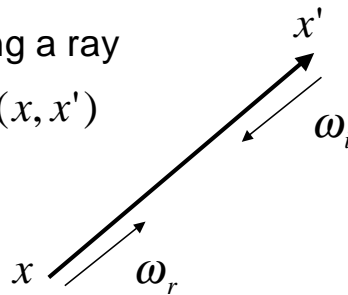
Deal directly with interaction with the environment

How does one surface affect another?

Radiance is invariant along a ray

$$L_i(x', \omega_i) = L_r(x, \omega_r) V(x, x')$$

$V(x, x')$ is the visibility
from point x to x'



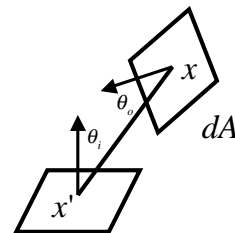
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Global Illumination

Converting the directional integral
into a surface integral

$$d\omega_i = \frac{\cos \theta_o dA}{|x - x'|^2}$$



The projected solid angle is:

$$\cos \theta_i d\omega_i = \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2} dA$$

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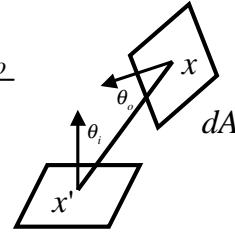
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Global Illumination

Geometry term:

$$G(x, x') = G(x', x) = \frac{\cos \theta_i \cos \theta_o}{|x - x'|^2}$$

$$\cos \theta_i d\omega_i = G(x, x') dA$$



Rewriting the reflectance equation:

$$L(x', \omega') = \int_S f_r(x, -\omega, \omega') L(x, \omega) V(x, x') G(x, x') dA$$

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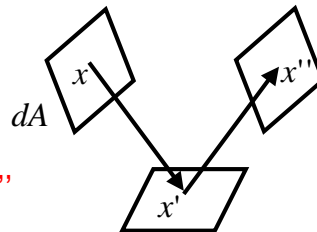
Global Illumination

Reparameterizing gives:

$$L(x', \omega') = \int_S f_r(x, -\omega, \omega') L(x, \omega) V(x, x') G(x, x') dA$$

$$L(x' \rightarrow x'') = \int_S f_r(x \rightarrow x' \rightarrow x'') L(x \rightarrow x') V(x, x') G(x, x') dA$$

The radiance sent from x' to x'' is simply the amount of radiance sent from all other visible points x in the scene and then reflected to x''



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Rendering Equation [Kajiya86]

Adding in the radiance directly emitted from x' to x'' yields the rendering equation:

$$L(x' \rightarrow x'') = L_e(x' \rightarrow x'') + \int_S f_r(x \rightarrow x' \rightarrow x'') L(x \rightarrow x') V(x, x') G(x, x') dA$$

The radiance sent from x' to x'' is simply the amount of radiance directly emitted from x' to x'' plus the radiance sent from all other visible points x in the scene and then reflected to x''

